# Multiple Scattering of Sound by Internal Waves and Acoustic Characterization of Internal Wave Fields in Deep and Shallow Water

Oleg A. Godin

CIRES/Univ. of Colorado and NOAA/OAR/Earth System Research Lab., R/PSD99 325 Broadway, Boulder, CO 80305-3328

phone: (303) 497-6558 fax: (303) 497-5862 email: oleg.godin@noaa.gov

Alexander G. Voronovich

NOAA/OAR/ Earth System Research Lab., R/PSD3, 325 Broadway, Boulder, CO 80305-3328 phone: (303) 497-6464 fax: (303) 497-6181 email: alexander.voronovich@noaa.gov

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#### LONG-TERM GOALS

- To improve understanding of the effects of internal waves (IWs) on sound propagation underwater.
- To develop an empirical statistical description of IWs in shallow water.
- To measure acoustically path-averaged energy of IWs in the ocean and variations of the energy on time scales from hours to years.
- To develop a comprehensive model of the IW spectra in the deep ocean and their regional and temporal variability.

# **OBJECTIVES**

- 1. To extend the existing theory of 3-D and 4-D acoustic effects induced by IWs to the regime of strong sound scattering.
- 2. To model the frequency shift and spectrum broadening of CW sound scattered by spatially-diffuse, random IWs in deep and shallow water.
- 3. To develop a predictive model of the acoustic frequency evolution at sound scattering by a train of tidally-generated, nonlinear IWs in a coastal ocean.
- 4. To assess the feasibility of inverting a measured frequency spectrum of the sound emitted by a narrow-band source, for spatially-averaged parameters of the IW fields in shallow and deep water.
- 5. To develop a theoretical description and a computer model of the average acoustic field in a deep ocean in the presence of a statistical ensemble of IWs.

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Form Approved OMB No. 0704-0188 6. To perform numerical simulations of the inversion of the mean acoustic field for IW characteristics and determine optimal parameters of the corresponding field experiment.

#### **APPROACH**

Three complementary representations of the acoustic field have been used in this work, namely, the ray-theoretical description of the field, full-wave representation of the field in the normal-mode basis, and the parabolic approximation.

A simple, convenient, and computationally efficient description of forward sound scattering by 3-D inhomogeneous, time-dependent sound speed fluctuations can be obtained by using the ray perturbation theory (Godin et al., 2006). The theory accounts for the change in geometry of 4-D (i.e., space-time) rays due to small fluctuations in the sound speed, and allows one to express first- and second-order perturbations in travel time, arrival angles, pressure amplitude, and other acoustic observables as integrals of weighted environmental perturbations calculated along unperturbed rays. When IW-induced sound speed variations are viewed as random fluctuations superimposed on a deterministic background, the theory expresses statistical moments of acoustic observables in terms of integrals of appropriate statistical moments of environmental perturbations. The integrals are calculated along unperturbed rays.

For a full-wave analytical description of 3-D and 4-D acoustic effects in the strong perturbation regime, where the spatial displacements of acoustic trajectories due to IW-induced perturbations are comparable to or larger than the correlation length of the environmental perturbations, we have used Chernov's "local method" in conjunction with the Markov approximation. The method allowed us to evaluate second moments of the acoustic filed. With this technique, only smallness of scattering over the correlation radius of the inhomogeneities is required. This condition is fulfilled for practical situations and for frequencies on the order of a hundred Hertz, however it can be violated at higher frequencies. In this case, standard diagrammatic technique can be used for calculation of the average field. This approach requires fluctuations of the index of refraction to be Gaussian, which is a good approximation in many practical situations. The equation for the average acoustic field in the statistically homogeneous in horizontal plane stratified waveguide satisfies an integral-differential equation. The kernel of the integral operator is calculated as a power series in the standard deviation of the refraction index. In the lowest order (Bourret approximation), the kernel is proportional to the spectrum of fluctuations. This equation also determines specific "average field" modes which generally can be different from the standard acoustic modes.

The key individuals that have been involved in this work are Oleg A. Godin, Andrey A. Grachev, Liudmila E. Matrosova, Vladimir E. Ostashev, and Liudmila Ye. Zabotina, (CIRES/Univ. of Colorado and NOAA/ESRL), and Alexander G. Voronovich and Valery U. Zavorotny (NOAA/ESRL). Dr. Voronovich has been primarily responsible for developing full-wave theoretical descriptions of multiple scattering of sound by internal gravity waves. Dr. Grachev advised on modeling hydrodynamic processes in the marine boundary layer. Mrs. Zabotina and Mrs. Matrosova have been involved in numerical simulations of the acoustic field. Drs. Zavorotny and Ostashev contributed their expertise on waves in random media. Dr. Godin took the lead in the theory and modeling of the 3-D and 4-D effects in underwater sound propagation. Dr. Michael A. Wolfson (APL, Univ. of Washington), who is supported by ONR through a separate project, supplied hundreds of individually-

generated realizations of sound speed fields, including cross-range sound speed gradients, in the presence of random IWs with the Garrett-Munk spectrum.

# WORK COMPLETED

Massive Monte Carlo simulations of sound propagation through random internal wave fields have been performed within the ray approximation. The emphasis in the simulations has been on accurate and self-consistent modeling of both sound speed perturbations and sound speed gradients induced by random IWs with the Garrett-Munk spectrum and on quantifying three-dimensional acoustic effects caused by cross-range sound speed gradients. The simulations provide data for calculation of single-and two-point statistical moments of acoustical quantities associated with random horizontal refraction of underwater sound, for assessment of significance of the three-dimensional effects and the role of various environmental parameters, and for evaluation of accuracy of the previously developed theoretical approach, which allows direct calculation of statistical moments of acoustic observables. Results of the simulations, which are illustrated in the next section of the present report, and their implications are being summarized in a paper in preparation for submission to J. Acoust. Soc. Am.

Validity of approximating the water-air interface as a pressure-release boundary has been re-assessed. It has been found that the pressure-release model becomes inapplicable when a point sound source is located at a depth smaller than acoustic wavelength. Bulk of the acoustic energy radiated by a shallow, low-frequency, monopole underwater source is emitted into the air (Godin, 2007b; Godin, 2007f; Godin, 2007g). From a theoretical analysis of multiple scattering of sound by a statistically rough fluid-fluid interface it has been found that the rough ocean surface can support a surface acoustic wave, which attenuates exponentially with height in air and depth in water and is weakly attenuated with range. Contribution of the surface wave into the acoustic field of a shallow underwater sound source has been calculated (Fuks and Godin, 2007).

Phenomenon of wavefront stability in a three-dimensionally inhomogeneous ocean has been studied theoretically. It has been shown that, at propagation ranges that are large compared to the correlation length of the sound-speed perturbations, end points of rays launched from a point source and having a given travel time are scattered primarily along the wavefront corresponding to the same travel time in the unperturbed environment. The ratio of RMS displacements of the ray end points along and across the unperturbed wavefront increases with range as the ratio of ray length to the correlation length of environmental perturbations. An intuitive physical explanation of the theoretical results has been proposed. The relative stability of wavefronts compared to rays has been shown to follow from Fermat's principle and dimensional considerations (Godin, 2007e).

Theoretical methods of investigation of sound fields in an inhomogeneous ocean have been summarized in a book (Brekhovskikh and Godin, 2007). A book on historical development of underwater acoustics research and technology in Russia and the former Soviet Union has been finalized and prepared for publication (Godin and Palmer, 2007).

# **RESULTS**

In this work, IWs are modeled as a Gaussian random field with the Garrett-Munk spectrum. IW modal structure is calculated assuming an exponential buoyancy profile. IW-induced sound speed perturbations, including cross-range sound speed gradients, are superimposed on a stratified

background with a canonical sound speed profile (Fig. 1a). For the sound speed gradients to be finite, spatial scales included in the Garrett-Munk spectrum need to be bounded from below. The smallest horizontal scale was chosen to be 250 m. Monte Carlo simulations of IW fields were verified against an independent calculation of statistical moments of the sound speed gradients (Fig. 1b).

A ray code described in (Godin et al., 2006) was used to model 3-D sound propagation in the ocean with IW-induced sound speed perturbations. Results of Monte Carlo simulations of sound propagation are illustrated in Figs. 2-5. Assuming that a sound source is located on the axis of the unperturbed waveguide, acoustic rays are characterized by their launch grazing angle  $\chi$ . Random horizontal refraction of sound caused by IW-induced cross-range sound speed gradients manifests, in particular, in the bearing angle  $\psi$  perturbations (Figs. 2, 3, and 5a), which have a zero mean, and the ray travel time corrections  $\Delta T$  (Figs. 4 and 5b). When cross-range sound speed gradients are weak, sound propagates faster in an ocean with cross-range sound speed gradients than in an ocean with the same sound speed field in the source-receiver vertical plane and no cross-range sound speed gradients (Godin et al., 2006). Therefore, the travel-time correction due to horizontal refraction is always negative:  $\Delta T < 0$ .

Monte Carlo simulations confirm theoretically predicted magnitude as well as angular and range dependence of 3-D acoustic effects in deep water. Travel time bias due to internal wave-induced random horizontal refraction increases with range as range squared while bearing angle variance increases linearly with range. In agreement with theoretical predictions, IW-induced horizontal refraction is strongest for refracted rays with turning points close to the surface, where magnitude of the horizontal sound speed gradients is close to its maximum, and is less pronounced for steeper and shallower rays (Figs. 2 - 4). Results obtained with eigenrays and unconstrained rays are similar as long as the propagation range is large compared to the ray cycle length (skip distance). However, because of their greater number, unconstrained rays provide a denser sampling of the environment and smoother averaged angular dependencies, when the number of available IW realizations is finite.

To study effects of the sound-speed perturbations in the source-receiver vertical plane on horizontal refraction, sound propagation in the ocean with random IWs was compared to propagation in an ocean with the same cross-range sound speed gradients and no in-plane sound speed perturbations (cf. Figs. 2 and 3). It was found that in-plane environmental perturbations, despite their strong influence on ray geometry in the source-receiver vertical plane and a possibility of ray chaos, have little statistical effect on horizontal refraction of steep rays. Scattering in the vertical plane increases horizontal refraction of shallow rays by enabling their access to shallow water where cross-range sound speed gradients are larger than near the waveguide axis.

PDFs of bearing angle fluctuations are found to be Gaussian (Fig. 5a), as predicted by the theory, with variance  $\sigma_{\psi}^2$  being dependent on propagation range and the ray launch angle. The variance is proportional to the path-averaged internal wave energy. PDFs of travel time corrections due to horizontal refraction are close to log-normal ones (Fig. 5b). For instance, parameters of the best-fitting

log-normal distribution 
$$P(\Delta T) = (2\pi)^{-1/2} \frac{1}{s\tau} \exp\left(-\frac{1}{2s^2} \ln^2 \frac{\Delta T}{\tau_0}\right)$$
 in Fig. 5b are  $\tau_0 = 1.22$  ms and  $s = 1.25$ 

0.83. The long tail of the probability distribution appears to be related to the spatial correlation of cross-range sound speed gradients.

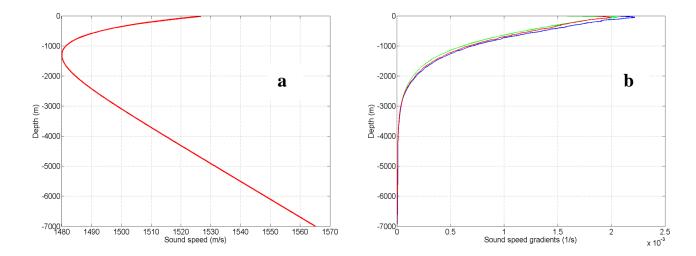


Figure 1. Average sound speed profile (a) and RMS horizontal sound gradients (b). Depth dependences of  $\left\langle \left(\partial c/\partial x\right)^2\right\rangle^{1/2}$  (blue) and  $\left\langle \left(\partial c/\partial y\right)^2\right\rangle^{1/2}$  (red) calculated from Monte Carlo simulations of internal wave fields agree well with the theoretical profile (green). [Sound speed has a single minimum of 1480 m/s at depth z=-1300 m and equals 1528 m/s and 1565 m/s at z=0 and z=-7000 m. RMS range- and cross-range components of the sound speed gradient have maxima of about  $2 \text{ s}^{-1}$  at  $z\approx-40$  m and decrease to zero with increasing depth.]

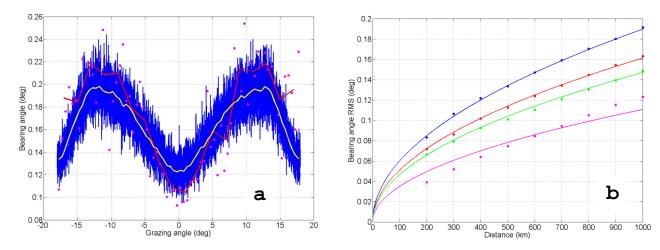


Figure 2. Dependence of RMS bearing fluctuations  $\sigma_{\psi} = \left\langle \psi^2 \right\rangle^{1/2}$  due to random internal gravity waves on launch angle of the ray (a) and on the propagation range (b). In Fig. 2a, four methods of  $\sigma_{\psi}$  calculation are distinguished by color: averages over 100 realizations of the internal wave field for 10000 unconstrained rays (blue), additional averaging over unconstrained rays in 0.5° intervals of the launch angle (yellow), and averages over the internal wave field realizations for eigenrays with additional averaging over 0.5° (magenta dots) and 2° (magenta line) intervals of the launch angle. Propagation range r = 1000 km. In Fig. 2b, the theoretical range dependence  $\sigma_{\psi} = \mathrm{const.} \cdot r^{1/2}$  (lines) is compared to Monte Carlo simulations of  $\sigma_{\psi}$  for unconstrained rays at various propagation ranges (dots) averaged over launch angle intervals (-1°, 1°) (magenta), (-5°, -3°) and (3°, 5°) (green), (-11°, -9°) and (9°, 11°) (blue), and (-18°, 18°) (red). [As a function of the launch angle of the ray,  $\sigma_{\psi}$  is minimum at  $\chi = 0$ , reaches maxima of about 0.2° at  $|\chi| \approx 13^{\circ}$  and decreases at larger  $|\chi|$ . Range dependence obtained in the Monte Carlo simulations closely follows the theoretically predicted one, especially at  $|\chi| \geq 3^{\circ}$ .]

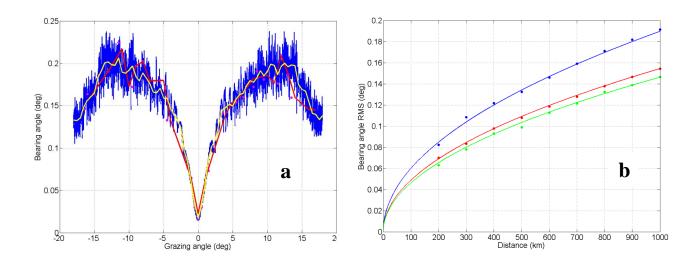


Figure 3. Same as Fig. 2 but without account for the sound speed fluctuations in the source-receiver vertical plane. [Graphs are similar to Fig. 2 except at small launch angles, where the minimum of  $\sigma_{_{\psi}}$  is about 0.02° and is much deeper in Fig. 3a than in Fig. 2a.]

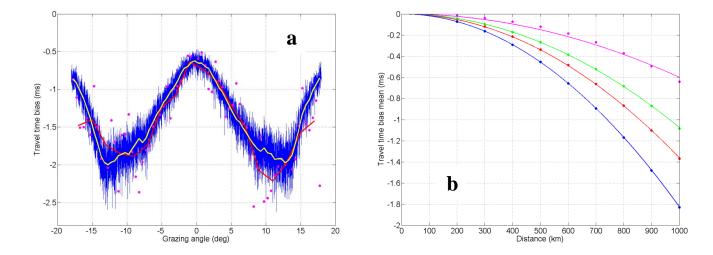
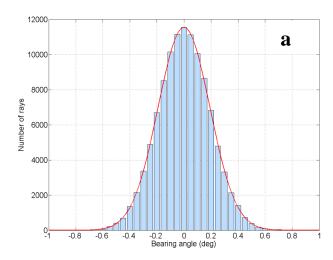


Figure 4. Dependence of the ray travel time bias  $\Delta T$  due to horizontal refraction caused by random internal gravity waves, on launch angle of the ray (a) and on the propagation range (b). In Fig. 4a, four methods of  $\Delta T$  calculation are distinguished by color: averages over 100 realizations of the internal wave field for 10000 unconstrained rays (blue), additional averaging over unconstrained rays in 0.5° intervals of the launch angle (yellow), and averages over the internal wave field realizations for eigenrays with additional averaging over 0.5° (magenta dots) and 2° (magenta line) intervals of the launch angle. Propagation range r=1000 km. In Fig. 4b, the theoretical range dependence  $\Delta T={\rm const...} r^2$  (lines) is compared to Monte Carlo simulations of  $\Delta T$  for unconstrained rays at various propagation ranges (dots) averaged over launch angle intervals (-1°, 1°) (magenta), (-5°, -3°) and (3°, 5°) (green), (-11°, -9°) and (9°, 11°) (blue), and (-18°, 18°) (red). [ $\Delta T$  is negative. As a function of the launch angle of the ray,  $|\Delta T|$  has a minimum of about 0.6 ms at  $\chi=0$ , reaches maxima of about 2.0 ms at  $|\chi|\approx 13^\circ$  and decreases at larger  $|\chi|$ . Range dependence obtained in the Monte Carlo simulations closely follows the theoretically predicted one.]



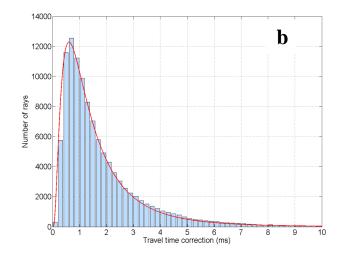


Figure 5. Histograms of the bearing angle perturbation (a) and the travel time correction due to horizontal refraction (b) at r = 1000 km for unconstrained rays with launch angles in the interval  $(-12^{\circ}, -8^{\circ})$  and their approximation (red lines) by a Gaussian (a) and log-normal (b) probability density functions. [The number of rays in 0.04 ° bins ranges from about 12000 at maxima of respective distributions to zero. The histograms for the bearing and travel time perturbations are closely approximated by the Gaussian and log-normal distributions, respectively.]

# IMPACT/APPLICATIONS

Results of Monte Carlo simulations confirm theoretical estimates of acoustic field fluctuations due to IW-induced cross-range sound speed gradients. The theoretical estimates can now be used to evaluate the feasibility of acoustic monitoring and devise an observation scheme to characterize IW fields in the ocean and their variability in geotime using measurements of acoustic fluctuations caused by random horizontal refraction.

# RELATED PROJECTS

Coherence of low-frequency sound signals propagating through a fluctuating ocean: Analysis and theoretical interpretation of 2004 NPAL experimental data (N00014-05-IP2-0024).

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